

GLOBALLY HYPERBOLIC GEODESICALLY COMPLETE COSMOLOGICAL MODEL

Leonardo Fernández Jambrina

*Dep. Enseñanzas Básicas,
 ETSI Navales,
 Avenida Arco de la Victoria s/n,
 E-28040-Madrid, Spain*

In this talk we shall show a perfect fluid cosmological model and its properties. The model possesses an orthogonally transitive abelian two-dimensional group of isometries that corresponds to cylindrical symmetry. The matter content is a stiff fluid that satisfies the energy and generic conditions. The metric is not separable in comoving coordinates for the fluid. The curvature invariants are shown to be regular everywhere in the coordinate chart and also indicate that the spacetime is asymptotically flat. Furthermore the causal geodesics are studied in order to determine that they are complete and that the model is globally hyperbolic. The model goes through an initial contracting epoch that is followed by an expanding era.

1 History

In 1990 it was published the first known non-singular cosmological solution of the Einstein equations for a perfect fluid with physically reasonable properties ¹. Later on it was proven that this solution was geodesically complete and globally hyperbolic as well as the absence of causally trapped sets ². These results triggered research on regular cosmological exact solutions.

A larger family of separable diagonal orthogonally transitive commuting G_2 metrics was found ³ and included in a general metric with FLRW cosmologies ⁴. Furthermore non-diagonal non-singular cosmological models have been shown ^{5, 6, 7} and also non-separable regular solutions can be found in the literature ⁸.

2 Another non-singular model

In this talk a cosmological model with cylindrical symmetry for a stiff perfect fluid is presented. It is non-singular and it is non-separable in comoving coordinates. Its metric tensor,

$$ds^2 = -\theta^0 \otimes \theta^0 + \theta^1 \otimes \theta^1 + \theta^2 \otimes \theta^2 + \theta^3 \otimes \theta^3, \quad (1)$$

which has been written in terms of an orthonormal coframe,

$$\theta^0 = e^{\frac{1}{2}K(t,r)} dt, \quad \theta^1 = e^{\frac{1}{2}K(t,r)} dr, \quad \theta^2 = e^{-\frac{1}{2}U(t,r)} dz, \quad \theta^3 = e^{\frac{1}{2}U(t,r)} r d\phi, \quad (2)$$

$$K(t, r) = \frac{1}{2} \beta^2 r^4 + (\alpha + \beta) r^2 + 2 t^2 \beta + 4 t^2 \beta^2 r^2, \quad (3)$$

$$U(t, r) = \beta (r^2 + 2 t^2), \quad \alpha, \beta \geq 0 \quad (4)$$

has been expressed in a chart where the coordinates ϕ and z are adapted to the Killing fields and the parametrization of the metric on the subspaces orthogonal to the group orbits is isothermal. The time coordinate ranges from minus infinity to infinity. The other coordinates have the usual range for cylindrical symmetry.

The metric is generically Petrov type I and does not admit further non-trivial isometries. The gradient of the transitivity surface element is spacelike as in every other non-singular model.

The expressions for the pressure and the density of the fluid,

$$p = \mu = \alpha e^{-K(t,r)}, \quad (5)$$

show that the Ricci curvature scalars are regular everywhere in the chart.

In the natural orthonormal coframe the kinematical quantities of the fluid read

$$u = \theta^0, \quad a = r \left(\beta^2 r^2 + \alpha + \beta + 4 \beta^2 t^2 \right) e^{\frac{1}{2} K(t,r)} \theta^1, \quad \omega = 0, \quad (6)$$

$$\begin{aligned} \sigma = \frac{4}{3} \beta t e^{-\frac{1}{2} K(t,r)} \{ & (1 + 2 \beta r^2) \theta^1 \otimes \theta^1 - \\ & -(2 + \beta r^2) \theta^2 \otimes \theta^2 + (1 - \beta r^2) \theta^3 \otimes \theta^3 \}, \end{aligned} \quad (7)$$

$$\Theta = 2 \beta t (1 + 2 \beta r^2) e^{-\frac{1}{2} K(t,r)}. \quad (8)$$

The deceleration parameter for this cosmology⁹ can be calculated from the expansion of the fluid. It shows that this universe undergoes a finite inflationary phase from $-\tau$ to τ ,

$$\tau(r) = \left[\frac{3}{4 \beta (1 + 2 \beta r^2)} \right]^{\frac{1}{2}}, \quad (9)$$

as it happens in other non-singular cosmological models.

3 Physical properties of the model

It has already been shown that the Ricci curvature scalars are regular. Furthermore all the curvature scalars are regular since it can be checked that the components of the Weyl tensor in a complex orthonormal coframe¹⁰ are bounded in the chart. They tend to zero for large values of the time and radial coordinates. Hence the universe is asymptotically Minkowski spacetime. The universe contracts until $t = 0$ and then begins to expand.

The matter content of the universe is a stiff perfect fluid with positive density everywhere in the chart. This means that both the strong and dominant energy conditions¹¹ and the generic condition¹² are fulfilled.

Since the most common definition of a spacetime singularity that is found in the literature is the existence of a causal geodesic which is incomplete, it will be necessary to study the geodesic equations for this model. The work is simplified by the existence of two independent constants of motion, which are related to translations along the z -axis and rotations around it.

The system of five second order geodesic equations in the affine parametrization can be shown to reduce to three first order equations plus two quadratures ¹⁰. From this system of equations a reasoning can be devised to establish that the range of the affine parameter of every causal geodesic is the real line. Therefore the spacetime is causally geodesically complete.

Concerning the causality conditions it is easy to notice that the model is causally stable since the coordinate function t is a cosmic time ¹¹. From the null geodesic completeness a stronger condition, global hyperbolicity, follows.

According to the singularity theorems ^{11, 12} this spacetime cannot contain trapped sets.

Acknowledgments

The present work has been supported by Dirección General de Enseñanza Superior Project PB95-0371 and by a DAAD (Deutscher Akademischer Austauschdienst) grant for foreign scientists. The author wishes to thank Prof. F. J. Chinae and Dr. L. M. González-Romero for valuable discussions and Prof. Dietrich Kramer and the Theoretisch-Physikalisches Institut of the Friedrich-Schiller-Universität-Jena for their hospitality.

References

1. J. M. M. Senovilla, Phys. Rev. Lett. **64**, 2219 (1990).
2. F. J. Chinae, L. Fernández-Jambrina, J. M. M. Senovilla, Phys. Rev. **D45**, 481 (1992) [arXiv: gr-qc/0403075]
3. E. Ruiz, J. M. M. Senovilla, Phys. Rev. **D45**, 1995 (1992).
4. J.M.M. Senovilla, Phys. Rev. **D53**, 1799 (1996).
5. M. Mars, Phys. Rev. **D51**, R3989 (1995).
6. J. B. Griffiths, J. Bičak, Class. Quantum Grav. **12**, L81 (1995).
7. M. Mars, J.M.M. Senovilla, Class. Quantum Grav. **14**, 205 (1997).
8. M. Mars, Class. Quantum Grav. **12**, 2831 (1995).
9. G. F. R. Ellis in *General Relativity and Cosmology, XLVII Enrico Fermi Summer School Proceedings*, ed. R. K. Sachs (Academic Press, New York, 1971).
10. L. Fernández-Jambrina, Class. Quantum Grav. **14**, 3407 (1997) [arXiv: gr-qc/0404017]
11. S. W. Hawking, G. F. R. Ellis, *The Large Scale Structure of Space-time*, (Cambridge University Press, Cambridge, 1973).
12. J. Beem, P. Ehrlich, *Global Lorentzian Geometry*, (Dekker, New York, 1981).